Functional Programming -- Mathematical Foundations

- **Simple Functions**
  - **function definition -- usual form:**

    cube(x) ≡ x * x * x, where x is a real number

    then, e.g., cube(2) = 8
  - **However, defining a function and naming the function are conceptually distinct**
  - **Lambda notation (Alonzo Church, 1941) provides for nameless functions:**

    λ(x) x * x * x
  - **Can apply just like a named function:**

    (λ(x) x * x * x) (2) = 8

Functional Programming -- Mathematical Foundations

- **Functional Forms**
  - ***function composition*: has two function parameters, yields a function whose value is the first function applied to the result of the second**

    $h = f \circ g$ -- means apply g first, then apply f to the result

    example: if $f(x) = x + 2$

    $g(y) = 3 * y$

    then $h(z) = f(g(z)) = (3 * z) + 2$

  - ***construction*: takes a list of functions and applies each in turn to the argument, creating a list of results**

    written by enclosing function names in brackets, e.g. [g,h,i]

    example: if $g(x) = x * x$

    $h(x) = 2 * x$

    $i(x) = x / 2$

    then [g, h, i] (4) yields (16, 8, 2)

  - ***apply-to-all*: takes a single function and applies it to a list of arguments, creating a list of values**

    denoted by α

    example: if $h(x) = x * x$

    then α(h, (2, 3, 4)) yields (4, 9, 16)

Functional Programming

- **LISP: John McCarthy 1958 MIT**
  - → **List Processing => Symbolic Manipulation**

- **Data Types**
  - → **Atoms : identifiers, symbols, numbers**
  - → **Lists (Sexpressions)**

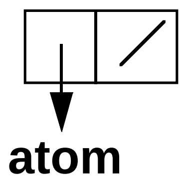
    (a b c d)

    (a (b c) d e)

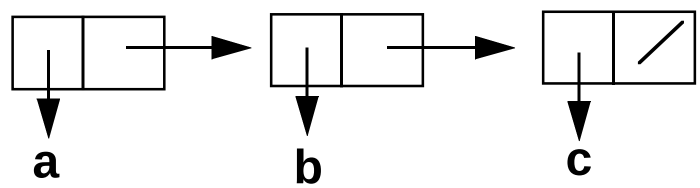
- **All data structures are single-linked nodes where each node has 2 pointers and represents a list element.**

Data Structures

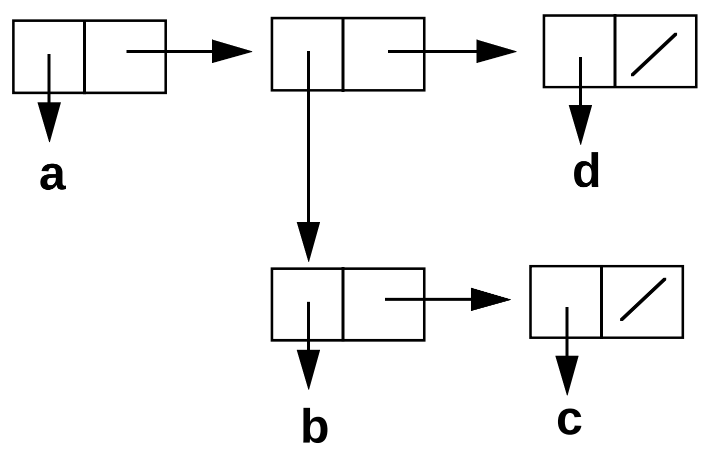
- **Single atom:**



- **List of atoms: (a b c)**



- **List containing list: (a (b c) d)**



LISP Primitives

- **quote => '**

  (quote a) => 'a = a

  (quote (a b c)) => '(a b c) = (a b c)

- **car : List => Sexp**
  - **One input arg => List**
  - **Returns first element of that list**

  (car '(a b)) => a

  (car '((a b) c)) => (a b)

  (car '(a (b c))) => a

  (car 'a) => undefined

  (car '()) => undefined

LISP Primitives

- **cdr : List => List**
  - **One input arg => List**
  - **Returns list of all elements but the first element**

    (cdr '(a b c)) => (b c)

    (cdr '((a b) (c))) => ((c))

    (cdr '(a)) => ()

    (cdr 'a) => undefined

    (cdr '()) => undefined

    (cdr '55) => undefined

LISP Primitives

- **cons : Sexp X List => List**

  → **2 args as input: (cons a1 a2)**

  a1 : Sexp

  a2 : List

  → **Returns a2 with a1 inserted as its first element**

  (cons 'a '(b c)) => (a b c)

  (cons 'a '()) => (a)

  (cons '(a b) '((c d) e)) => ((a b) (c d) e)

  → **Be careful, cons will take non list (atom) arguments and form "dotted" pairs, e.g.**

  (cons 'a 'b) => (a . b)

  car **cdr**

LISP Predicates

- **Predicates are functions that return true (#t) or false (nil ())**
  - → **the following return #t if the arguments are of the indicated type, and nil () otherwise**

```
  (symbol?  'a)      =>    #t
  (symbol?  '())     =>    ()

  (number? '55)      =>    #t
  (number?  55)      =>    #t

  (atom?  'a)        =>    #t
* (atom?  '())       =>    #t
  (atom?  '(a))      =>    ()

  (list?  '(a))      =>    #t
  (list?  'a)        =>    ()
* (list?  '())       =>    #t

  (null?  '())       =>    #t
  (null?  '(a c))    =>    ()
```

Additional Functions

- **eq?   Sexp X Sexp   => { #t , () }**
  - **Returns true if objects are equal through pointer comparison. Guaranteed to work on symbols**
- **equal?   Sexp X   Sexp   =>   {#t , () }**
  - **Recursively compares two objects to determine if they are equal (works on symbols, atoms, numbers, and lists.**
- **= , < , >   number X   number   => { #t, () }**
  - **Performs numeric comparison on two numbers**
- **+ , - , * , /   number X number   =>   number**
  - **Performs designated numeric operation**
  - **Scheme provides exact and inexact numbers**

Control Flow

- **cond : (predicate Sexp)$^+$ => eval (Sexp)**
  - → **Evaluates (predicate Sexp) pairs.**
  - → **Each predicate is evaluated in sequence until one is found to be true. The corresponding evaluated Sexp is returned.**

```
(cond
   (Pred  Sexp)
   (Pred  Sexp)
        :
   (Pred  Sexp)
   (else Sexp)
)
```

```
(cond
   ((null? lis1) lis2)
   ((atom? (car lis1)) (car lis1))
   (else (cdr lis1))
)
```

Additional Functions

- **Additional control flow primitives that are available:**

  (if Sexp$_1$ Sexp$_2$ [Sexp$_3$])

  if (Sexp$_1$) then Sexp$_2$ [else Sexp$_3$]

  (while Sexp$_1$ Sexp$_2$)

  while (Sexp$_1$) do (Sexp$_2$) od

- **Blocking primitive:**

  (begin Sexp$_1$ Sexp$_2$ ... Sexp$_n$)

- **Variable initialization primitive:**

  (set! x Sexp)

- ***USE THESE FOR TESTING PURPOSES ONLY!***

Function Definition

```
(define (fctn_name arg1  arg2  ... argi  )
    Sexp
)

(define (atom? atm)
  (cond
    ((list? atm) (null? atm))
    ((symbol? atm) #t)
    (else ())
  )
)

(define (equal? lis1 lis2)
  (cond
    ((atom? lis1) (eq? lis1 lis2))
    ((atom? lis2) ())
    ((equal? (car lis1) (car lis2))
      (equal? (cdr lis1) (cdr lis2)))
    (else ())
  )
)
```

Function Definition

```
(define (member? atm lis)
  (cond
    ((null? lis) ())
    ((eq? atm (car lis)) #t)
    (else (member? atm (cdr lis)))
  )
)

(define (fac n)
  (cond
    ((eq? n 0) 1)
    (else (* n (fac (- n 1))))
  )
)

(define (append lis1 lis2)
  (cond
    ((null? lis1) lis2)
    (else (cons (car lis1)
        (append (cdr lis1) lis2)))
  )
)
```

Lambda Expressions

- **Intuitively, *lambda expressions* allow one to define and use nameless functions and to pass them to be used in other functions**

```
(lambda (lis) (car (cdr lis)))
```

- **Given to a lisp interpreter, the above function definition returns the second element in a list, e.g.**

```
((lambda (lis) (car (cdr lis))) '(a b c))
```

returns "b"

Lambda Expressions

- **We CAN integrate the lambda expression into a function definition:**

```
(define second
        (lambda (lis) (car (cdr lis)))
)
```

  - **Once "evaled" by the interpreter, the function definition is "bound" to the name "second" such that**

```
(second '(a b c))  =>  b
```

  - **But, our "standard" way of defining functions will work too ...**

```
(define (second lis)
   (car (cdr lis))
)

(second '(a b c))  => b
```

  - ***SO, WHAT DOES THE LAMBDA EXPRESSION BUY US?***

Lambda Expressions

- **WE NOW HAVE THE CAPABILITY TO *PASS FUNCTION DEFINITIONS AS PARAMETERS!***
  - **Suppose that we want to write an "apply-to-all" function that takes a function definition and list as its arguments and applies its function argument to all elements in the list.**

```
(define (mapcar fctn lis)
  (cond
    ((null? lis) ())
    (else (cons (fctn (car lis))
             (mapcar fctn (cdr lis))))
  )
)
```

  - **then**

```
(mapcar(lambda(num)(* num num)) '(2 4 6))
```

returns a list containing the square of all elements in the original list, i.e.,

(4 16 36)

Lambda Expressions

- **Why not simply define a function “f” that performs a specified operation on one element and then pass IT to mapcar, e.g.**

```
(define (square x)
   (* x x)
)
(define (mapcar fctn lis)
   (cond
      ((null? lis) ())
      (else (cons (fctn (car lis))
           (mapcar fctn (cdr lis))))
   )
)
```

→ **and then....**

```
(mapcar square '(2 4 6))  ????
```

Scoping in LISP

- **In reality, Lisp does allow one to define global and local variables, e.g.**

```
(define x 5)                 ; Global x
(set! x (car '(a b c))       ; Gbl/Lcl x
```

If (set! ...) *inside* a function defn

If (set! ...) *outside* a function defn

In reality, Lisp does allow programs to reference "unbound" variables, e.g.

```
(define (f atm)
  (cons x y)           ; y is an unbound variable
)                      ; x assumes the prev  (set! x.... )
```

→ **What are the implications of these capabilities with respect to scoping?**

Scoping in LISP

- **Consider the following example:**

```
(define (A ...)
   ... (car X) ...   ;      unbound ref
)


(define (B Fctn X)
   ... Fctn ...    ;        invoke Fctn *
)


(define (C X Z)
  ... A ...       ;         invoke fctn A  **
  ... (B A Z) ... ;         invoke fctn B  ***
)
```

Scoping in LISP

- **Assuming that "our" Lisp is *statically* scoped (and most current Lisps are statically scoped), let's consider the impact of the following invocation of C:**

  (C ‘(i j) ‘(k l m))

  - **What is the binding of X in A after being invoked at ** ?**
  - **What is the binding of X in A after being sent to B through the call *** and being invoked at * ?**
  - **Is it what you expected?**

- **What is needed is the ability to bind an *execution environment* at same time a function is passed as a parameter! (Funarg Problem)**

  - **Solution: ... (B (function A) Z) ...**

SCOPING IN LISP

- **Consider the same scenario again:**

```
(define (A ...)
   ... (car X) ...
)

(define (B Fctn X)
   ... Fctn ...
)

(define (C X Z)
   ... A ...
   ... (B (function A) Z) ...
)
```

- **In Pascal, static scoping and lexical scoping are effectively synonymous.**

  → **Although we are able to achieve "static" scoping through the use of the *function* primitive, is this also a "lexical" scoping?**

XSCHEME

- **XSCHEME is a <u>dialect</u> of LISP**

- **XSCHEME extends LISP**
  - → **Minor syntax changes**
    - will not affect us
  - → **Has extensions**
    - additional functions
      we will not use

- **We will use only the “pure” LISP parts of XSCHEME**

XSCHEME: Helpful Hints

- **Place "(exit)" at end of file**
- **Run "xscheme < infile > outfile"**
- **For debugging purposes only:**
  - **(if (< x y) x**

    (if ....

```
(begin
    (write "x is")
    (write x)
    (if (< x y ) x
    (if .....
                 ) ; for begin
```

Allows you to display intermediate computations